

Instructions: Complete each of the following exercises for practice.

1. Compute the length of the give curve.

(a) $\mathbf{r}(t) = \langle t, 3 \cos(t), 3 \sin(t) \rangle, -5 \leq t \leq 5$

(d) $\mathbf{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j} + \ln(\cos(t))\mathbf{k}, 0 \leq t \leq \frac{\pi}{4}$

(b) $\mathbf{r}(t) = \langle 2t, t^2, \frac{1}{3}t^3 \rangle, 0 \leq t \leq 1$

(e) $\mathbf{r}(t) = \mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}, 0 \leq t \leq 1$

(c) $\mathbf{r}(t) = \sqrt{2}t\mathbf{i} + e^t\mathbf{j} + e^{-t}\mathbf{k}, 0 \leq t \leq 1$

(f) $\mathbf{r}(t) = t^2\mathbf{i} + 9t\mathbf{j} + 4t^{\frac{3}{2}}\mathbf{k}, 1 \leq t \leq 4$

2. Parameterize the curve $\mathbf{r}(t)$ with respect to arc length measured from P in the direction of increasing t .

(a) $\mathbf{r}(t) = \langle 5 - t, 4t - 3, 3t \rangle, P = (4, 1, 3)$

(b) $\mathbf{r}(t) = \langle e^t \sin(t), e^t \cos(t), \sqrt{2}e^t \rangle, P = (0, 1, \sqrt{2})$

3. Reparametrize the curve $\mathbf{r}(t) = \frac{1-t^2}{1+t^2}\mathbf{i} + \frac{2t}{t^2+1}\mathbf{j}$ with respect to arc length measured from the point $(1, 0)$ in direction of increasing t ; simplify where possible. What can you conclude about the curve?

4. Compute unit tangent and unit normal vectors to $\mathbf{r}(t) = \langle t, 3 \cos(t), 3 \sin(t) \rangle$; compute the curvature $\kappa(t)$.

5. Compute the curvature $\kappa(t)$ for the curve $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + e^t\mathbf{k}$.

6. Compute the curvature $\kappa(t)$ for $\mathbf{r}(t) = \langle t^2, \ln(t), t \ln(t) \rangle$; what is the curvature at the point $P = (1, 0, 0)$?

7. Compute the unit tangent, unit normal, and binormal vectors of $\mathbf{r}(t) = \langle \cos(t), \sin(t), \ln(\cos(t)) \rangle$.

8. Compute equations of the normal and osculating planes of the given curve at the given point.

(a) $x = \sin(2t), y = -\cos(2t), z = 4t$ at $(0, 1, 4\pi)$

(b) $x = \ln(t), y = 2t, z = t^2$ at $(0, 2, 1)$

9. Prove that $\frac{\partial \mathbf{T}}{\partial s} = \kappa \mathbf{N}$ for all curves $\mathbf{r}(t)$.